MULTIMEDIA UNIVERSITY

STUDENT ID NO | | | | | | | | | | |

MULTIMEDIA UNIVERSITY

FINAL EXAMINATION

TRIMESTER 3, 2021/2022

TMA1201 – DISCRETE STRUCTURES AND PROBABILITY

(TC1V / TT1L)

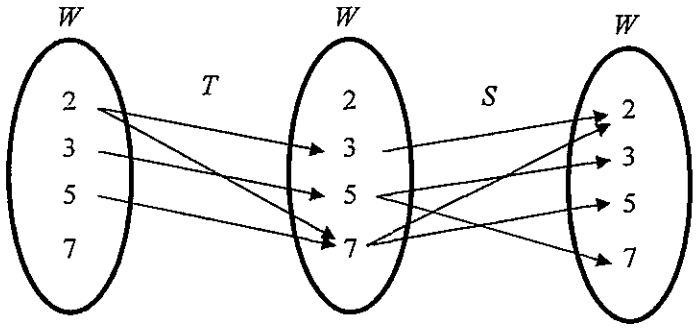
3 OCTOBER 2022
9 a.m. – 11 a.m
(2 Hours)

INSTRUCTIONS TO STUDENTS

1. This Question paper consists of **5** printed pages (inclusive of the cover page).
2. Answer **ALL** questions. The distribution of marks for each question is given.
3. Please write all your answers in the Answer Booklet provided.
4. Please write the answers for each question on a new page.
5. An Appendix page is provided as the last printed page of this Question paper.

Question 1 (25 marks)

a) An arrow diagram that shows a composition of relation is shown in the diagram below.



(i) List the elements of the relation set. [1 mark]

(ii) Determine if the relation is reflexive, symmetric, antisymmetric, and/or transitive. Explain your answers. [4 marks]

(iii) List the elements of the relation S^{-1}. [1 mark]

(iv) Is the relation S^{-1} a function? Explain. [1 mark]

b) Given the following predicates.

$S(x)$: x is a new smartphone model

$V(x)$: x is expensive

(i) Translate the following predicate statements using logical connectives and quantifiers, wherever required.

Every new smartphone model is expensive.

Any old model of smartphone is cheap.

A folding phone is a new smartphone model.

Thus, there is a smartphone model that is new. [4 marks]

(ii) Apply formal reasoning using the Rules of Inference to prove the arguments in (i). Let the Universe of Discourse (UOD) be all smartphones in the world. [4 marks]

Continue...

c) Given that $P(x,y): y+\frac{1}{x}=x$, where $x,y \in \mathbb{Z}$. Determine the truth value of the following. Explain your answers.

(i) $P(1,-1)$ [1 mark]

(ii) $\exists y P(1,y)$ [1 mark]

(iii) $\forall x \exists y P(x,y)$ [1 mark]

d) Use mathematical induction to prove the following mathematical statement,

$$P(n): 2^n > n+1 \qquad n>1$$ [5 marks]

e) Show that the $f(x)=\frac{2}{x}+x$ has a complexity function, $O(n)$. [2 marks]

Question 2 (25 marks)

a) Given the following adjacency matrix.

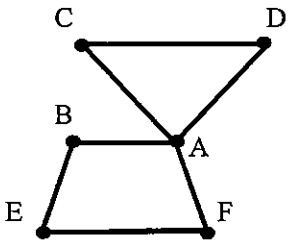
$$\begin{bmatrix} 1 & 2 & 2 & 1 \\ 2 & 2 & 0 & 1 \\ 2 & 0 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{bmatrix}$$

(i) Draw the corresponding graph. Label the vertices as **W, X, Y, Z.** [2 marks]

(ii) Calculate the degree for each vertex, **W, X, Y, Z.** [2 marks]

(iii) Determine if the Graph has a Euler Circuit and/or Euler Trail. Explain your answers. [2 marks]

b) Given the following graph,



Create a spanning tree using the breadth-first search (BFS) algorithm with **A** as the root. Note that alphabetical ordering applies when selecting a node. [6 marks]

Continue...

c) Given the following state transition table for a finite-state automaton (FSA) of $S = \{S_0, S_1, S_2\}$, $I = \{a, b, c\}$, $A = \{S_1\}$, and initial state, S_0.

	Next State		
Inputs / States	a	b	c
S_0	S_0	S_1	S_2
S_1	S_0	S_1	S_2
S_2	S_2	S_2	S_2

(i) Create the state-transition diagram for the FSA. [3 marks]

(ii) Give an example of a 4-bit string that will be accepted by the FSA. [1 mark]

(iii) State the language recognises by the FSA. [2 marks]

d) In how many ways can letters of the word below be arranged if,

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(i) there are no restrictions? [1 mark]

(ii) all the letters 'E' must be together? [1 mark]

e) A cake shop sells various types of dessert. $P(X = x)$ represents the probability that a random X types of dessert will be bought in a single day at a cake shop. The probability distribution (pdf) for X is shown below.

X	0	1	2	3	4
$P(X=x)$	0.13	0.24	a	0.17	0.15

(i) Find the value of a. [1 mark]

(ii) Calculate the probability that a customer will buy between 1 and 3 types of dessert at the cake shop. [1 mark]

(iii) Calculate the expected value, $E[X]$ for the pdf. [1 mark]

(iv) Given that when a customer buys more than 2 types of dessert is a success, calculate the probability that in a sample of 10 customers, exactly 9 customers buy more than 2 types of dessert in a day. [2 marks]

End of Questions.

APPENDIX

Name	Tautological Arguments	Premises	Conclusion
Addition	$p \Rightarrow p \vee q$	1. p	$p \vee q$
Simplification	$(p \wedge q) \Rightarrow p$	1. p 2. q	p
Modus Ponens	$[p \wedge (p \rightarrow q)] \Rightarrow q$	1. p 2. $p \rightarrow q$	q
Modus Tollens	$[\neg q \wedge (p \rightarrow q)] \Rightarrow \neg p$	1. $\neg q$ 2. $p \rightarrow q$	$\neg p$
Hypothetical Syllogism	$[(p \rightarrow q) \wedge (q \rightarrow r)]$ $\Rightarrow (p \rightarrow r)$	1. $p \rightarrow q$ 2. $q \rightarrow r$	$p \rightarrow r$
Disjunctive Syllogism	$[(p \vee q) \wedge \neg p] \Rightarrow q$	1. $p \vee q$ 2. $\neg p$	q
Resolution	$[(p \vee q) \wedge (\neg p \vee r)]$ $\Rightarrow (q \vee r)$	1. $p \vee q$ 2. $\neg p \vee r$	$q \vee r$
Conjunction	$p \wedge q \Rightarrow (p \wedge q)$	1. p 2. q	$p \wedge q$

Identity laws $p \wedge T \Leftrightarrow p$ $p \vee F \Leftrightarrow p$	**Associative laws** $(p \vee q) \vee r \Leftrightarrow p \vee (q \vee r)$ $(p \wedge q) \wedge r \Leftrightarrow p \wedge (q \wedge r)$	**Conversion of Implication** $p \rightarrow q \Leftrightarrow \neg p \vee q$ $p \vee q \Leftrightarrow \neg p \rightarrow q$
Domination laws $p \vee T \Leftrightarrow T$ $p \wedge F \Leftrightarrow F$	**Absorption Laws** $p \vee (p \wedge q) \Leftrightarrow p$ $p \wedge (p \vee q) \Leftrightarrow p$	**Contrapositive** $p \rightarrow q \Leftrightarrow \neg q \rightarrow \neg p$ $p \leftrightarrow q \Leftrightarrow \neg q \leftrightarrow \neg p$
Idempotent laws $p \vee p \Leftrightarrow p$ $p \wedge p \Leftrightarrow p$	**Commutative laws** $p \vee q \Leftrightarrow q \vee p$ $p \wedge q \Leftrightarrow q \wedge p$	**Conversion of Equivalence** $p \leftrightarrow q \Leftrightarrow (p \rightarrow q) \wedge (q \rightarrow p)$
Negation laws $p \vee \neg p \Leftrightarrow T$ $p \wedge \neg p \Leftrightarrow F$ **Double Negation laws** $\neg(\neg p) \Leftrightarrow p$	**Distributive laws** $p \vee (q \wedge r) \Leftrightarrow (p \vee q) \wedge (p \vee r)$ $p \wedge (q \vee r) \Leftrightarrow (p \wedge q) \vee (p \wedge r)$ $(p \vee q) \wedge r \Leftrightarrow (p \wedge r) \vee (q \wedge r)$ $(p \wedge q) \vee r \Leftrightarrow (p \vee r) \wedge (q \vee r)$	**De Morgan's laws** $\neg(p \wedge q) \Leftrightarrow \neg p \vee \neg q$ $\neg(p \vee q) \Leftrightarrow \neg p \wedge \neg q$